Buoyancy-Driven Circulation in Bubble Columns: Alternative Analysis

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Two-phase flows in bubble columns are typically characterized by the buoyancy-driven liquid circulation, which is confirmed to be governed by the radial nonuniform distribution of the gas phase (Ueyama and Miyauchi, 1979; Anderson and Rice, 1989). One of the recent efforts on modeling and simulation of the liquid flow in bubble columns is directed to the 1-D flow models. examples of which may be found in Burns and Rice (1997), Geary and Rice (1992), Rice and Geary (1990), Anderson and Rice (1989), Kumar et al. (1994), Gharat and Joshi (1992), and Clark et al. (1987). A new analysis of this type of flow model is attempted here. In contrast to the previous modeling work, however, the present analysis focuses on the main phenomenological characteristics of the buoyancy-driven circulation instead of on predicting turbulent parameters and liquid velocity distributions (Burns and Rice, 1997; Geary and Rice, 1992). Through this new analysis, three main characteristic points of the flow (the reversion point of the flow, radial position of the local maximum Reynolds shear stresses, and maximum reverse flow position) are derived explicitly. Inherent relationships of these characteristic points are also detailed by the present theory, which enables flow parameter selection and evaluation in computations. Furthermore, the maximum Reynolds shear stress emerges for the first time as an important flow parameter characteristic of the flow structure in bubble columns.

Characteristics of the Flow: Theoretical Analysis vs. Modeling

Our analysis is based on the widely adopted governing equation of the 1-D flow models (Yang et al., 1986)

$$\frac{d[r\tau]}{rdr} = \frac{2}{R}\cdot\tau_w - \left(\overline{\epsilon_g} - \epsilon_g\right)\rho_l g \qquad (1)$$

where the gas holdup distribution can be fitted by the following relation (Gharat and Joshi, 1992; Yang et al., 1986; Ueyama and Miyauchi, 1979)

$$\epsilon_g(\xi) = \frac{m+2}{m}\cdot\overline{\epsilon}_g\cdot(1-\xi^m) \qquad (2)$$

Given the gas holdup profile, one may perform the kinematic analysis to predict the radial liquid velocities, as extensively reported in the bubble column literature (Burns and Rice, 1997). Alternatively, a dynamic analysis may provide unique insight into the buoyancy-driven circulation features in bubble columns, as will be shown.

Substituting Eq. 2 into Eq. 1, and introducing the following dimensionless variables

$$T = \tau/\tau_w \qquad Re_w = (|\tau_w|\rho_l)^{1/2}R/\mu_m \qquad (3)$$

where T is the dimensionless Reynolds shear stress, Re_w is the Reynolds number, and $\tau_w < 0$ to ensure the liquid circulation (Rice and Geary, 1990), and

$$\mathrm{Ar} = \frac{\rho_l^2 R^3 g\overline{\epsilon_g}}{\mu_m^2} \qquad \beta = \frac{Re_w^2}{\mathrm{Ar}} \qquad (4)$$

it is obtained that

$$\frac{d[\xi T]}{\xi d\xi} = 2 + \frac{1}{m\beta}[(m+2)\cdot\xi^m - 2] \qquad (5)$$

We assume the following force-continuity condition

$$\delta^+ T(\xi = \delta^+) = \delta^- T(\xi = \delta^-) \qquad (6)$$

for the zero velocity point at the radial position, $\xi = \delta$, that is, the so-called reversion point (Burns and Rice, 1997). Accordingly, with the left side of Eq. 5 being zero, the reversion point is given by

$$\delta = \left[\frac{2}{(m+2)}(1-m\beta)\right]^{1/m} \qquad (7)$$

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Integrating Eq. 5 results in an expression of the dimensionless local Reynolds shear stress as

$$T = -\frac{\xi}{m\beta}\cdot(1 - m\beta - \xi^m) \tag{8}$$

Using the maximum condition $dT/d\xi = 0$, we obtain the dimensionless maximum Reynolds shear point as

$$\gamma = \left[\frac{1}{(m+1)}(1 - m\beta)\right]^{1/m} \tag{9}$$

Experimental evidence (Moslemian et al., 1992; Devanathan et al., 1990) confirmed the presence of a local maximum Reynolds shear, which is rather typical for flow in bubble columns but different from that in two-phase pipe flows (Serizawa et al., 1975). However, this feature is seldom reported in previous theoretical analyses. This maximum condition is treated as a stress continuity condition in multi-zones models (Anderson and Rice, 1989; Rietema and Ottengraf, 1970). The difference between γ and δ will be discussed later.

By definition ($T = 0$), the maximum reverse flow velocity point is derived from Eq. 8

$$\lambda = (1 - m\beta)^{1/m} \tag{10}$$

Based on the preceding results, the following corollaries can be derived. These are

$$\tau_w = \frac{\rho_l g \overline{\epsilon_g} R}{m}(\lambda^m - 1) \tag{11}$$

$$\frac{\delta}{\lambda} = \left[\frac{2}{m+2}\right]^{1/m} \tag{12}$$

and

$$\frac{\gamma}{\lambda} = \left[\frac{1}{m+1}\right]^{1/m} \tag{13}$$

Substituting the alternative form of Eq. 11

$$1 - m\beta = \lambda^m \tag{14}$$

into Eq. 8 yields

$$T = -\frac{\lambda^m}{m\beta}\cdot\xi\cdot\left[1 - \left(\frac{\xi}{\lambda}\right)^m\right] \tag{15}$$

It should be noted that this expression holds throughout the flow cross section, especially including the region of negative shear stresses. This treatment can avoid the need to consider the complicated multi-zones method (Anderson and Rice, 1989; Rietema and Ottengraf, 1970). Similarly, the local maximum shear stress may be written as

$$T_{\max} = -\frac{1}{\beta}\cdot\left(\frac{\lambda^m}{m+1}\right)^{(m+1)/m} \tag{16}$$

Results and Discussion

Comparison of the theory with previous modeling work

To test the present theory, the numerical data of λ by Burns and Rice (1997) is selected as a known value to give the predicted δ, γ, and τ_w, using Eqs. 7, 9, and 11, respectively. The results are shown in Table 1. It is noted that values of smaller τ_w and larger δ than those of Burns and Rice (1997) are predicted by the present work, while our predicted λ approximates to the predicted values of δ by Burns and Rice (1997). The explanation of this trend can be attributed to the different dynamic conditions imposed at the reversion point and the gas holdup fitting relations adopted thereof. According to Burns and Rice (1997), an assumed three-zone pluglike gas holdup distribution may be written as

$$\overline{\epsilon_g} = \epsilon_c \delta^2 + \epsilon_B(\lambda^2 - \delta^2) \tag{17}$$

where ϵ_c and ϵ_B are the average holdups in the core and buffer regions respectively. Although two independent implicit kinematic constraints, continuity and zero velocity condition at $\xi = \delta$, have been imposed to specify λ and δ (Burns and Rice, 1997), it is found that the explicit relation between λ and δ, that is, Eq. 17, may provide the third constraint. The consistency of these three independent conditions is left unexplored in their article. Our computation showed that with the known data of λ by Burns and Rice (1997) the predicted values of δ from Eq. 17 are the same as theirs (Table 1) which are claimed to be jointly defined with λ by the first two constraints. It can be proved that Eq. 17 is an alternative form of the stress continuity condition between the core and buffer regions, therefore constituting the maximum Reynolds shear stress condition. Hence, the zero velocity condition is not necessary or may be self-consistent with the stress continuity condition. The approximation of our γ to Burns and Rice's δ originates from the same stress continuity condition imposed. In terms of the present analysis, the two points are different from each other, while Burns and Rice (1997) treated them as the same.

Table 1. Predicted and Computed Values of τ_w, γ, δ and λ*

Reference	U_g (cm/s)	D (cm)	m Fitting	$\overline{\epsilon_g}$ Exp.	Computed by Burns & Rice (1997) λ	δ	τ_w Pa	Predicted by This Work γ	δ	τ_w Pa
Hills (1974)	1.9	13.8	8	0.07	0.981	0.725	−0.926	0.745	0.802	−0.842
Hills (1974)	3.8	13.8	6	0.137	0.982	0.733	−1.714	0.710	0.779	−1.594
Hills (1974)	6.4	13.8	5	0.182	0.983	0.741	−2.147	0.686	0.765	−2.022
Devanathan et al. (1990)	10.5	29.2	6	0.180	0.988	0.726	−3.147	0.714	0.784	−3.000

*Burns and Rice (1997).

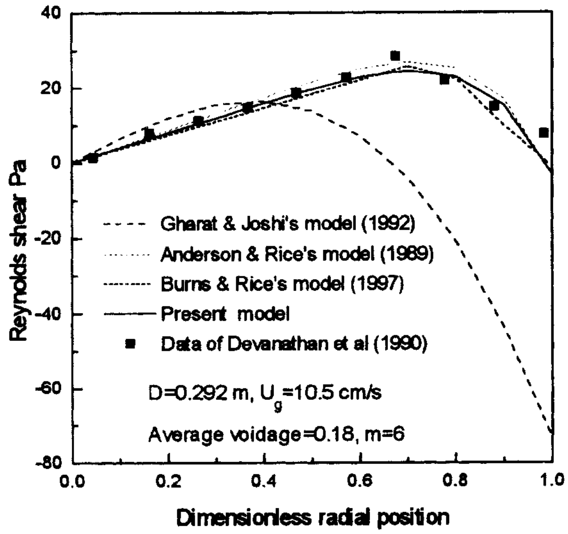

Figure 1. Reynolds shear stress: various model predictions vs. experimental data of Devanathan et al. (1990).

Different model results of the local Reynolds shear stresses are illustrated in Figure 1, together with the experimental data of Devanathan et al. (1990). In preparation of the figure, the present model result is obtained with Eq. 15 with known values of λ (= 0.988) and m (= 6.0) given by Anderson and Rice (1989). The phase slip velocity is estimated to be 0.25 cm/s in accordance with Gharat and Joshi's model (1992).

The model of Gharat and Joshi predicts a rather poor distribution of the local shear stress due to an overestimate of the wall shear, while the present model and the models of Rice et al. yield (Anderson and Rice, 1990; Burns and Rice, 1997) approximately the same results, which agree well with those from experiments. However, slight differences can be noted in predicting the maximum Reynolds shear stress points. The reason for this discrepancy may be attributed to the foregoing analysis.

Application of the theory to simplified cases for $\lambda \approx 1.0$

Physically, $\lambda \in [0,1]$; thus $\beta \in [1/m, 0]$. Equation 10 can therefore be simplified to be a series representation

$$\lambda = 1.0 - \beta + O\left(\frac{[1-m]}{2!}\beta^2\right) \tag{18}$$

As a first approximation, Eq. 18 may be written as

$$\lambda \approx 1.0 - \beta \tag{19}$$

This suggests that λ is not sensitive to the changes in m, and its value can be solely determined by β. For an inviscid large bubble column $\beta \ll 1.0$; hence $\lambda \approx 1.0$. Therefore

$$\delta = \left[\frac{2}{m+2}\right]^{1/m} \tag{20}$$

When $m = 2$, then $\delta = \sqrt{1/2} \approx 0.7$. This value is suggested by both experimental and theoretical works (Ueyama and Miyauchi, 1979; Yang et al., 1984; Burns and Rice, 1997). It is also noted that the circulation vanishes when $\delta \to 1$, which is the physical limit in the case of ideal bubbly flows (Burns and Rice, 1997).

Also with $\lambda \approx 1.0$, Eq. 16 becomes

$$\tau_{t,\max} = \rho_l g \overline{\epsilon_G} R \cdot \left(\frac{1}{m+1}\right)^{1+(1/m)} \tag{21}$$

This is a simplified expression of the maximum Reynolds stress, on which the effects of system parameters can be investigated. On the one hand, the average gas holdup is found to be correlated with a wide range of liquids by the relation (Kumar et al., 1997)

$$\frac{\overline{\epsilon_g}}{1-\overline{\epsilon_g}} = U_g^{0.7} \tag{22}$$

This means that changes in column diameter have slight effects on the mean gas holdup. On the other hand, the column radius is coupled with the distribution factor as a mixed factor affecting the maximum Reynolds stress. It is therefore argued that the maximum Reynolds stress may be estimated with known system parameters and the distribution factor or vice versa. There is sparse available experimental Reynolds stress data in the literature for verification of this. First, the computation examples of Rice and Geary (1990) as well as Geary and Rice (1992) are selected to give the predicted values of the maximum stresses, as shown in Table 2. From Table 2, trends of increase in the Reynolds stress but decrease in m with the superficial gas velocities are noted for both systems. The last line of the table shows that the predicted value of the maximum Reynolds shear stress is good.

Alternatively, the local Reynolds stress data of Moslemian et al. (1992) by PC-based computer automated radioactive particle tracking are used to estimate the gas holdup distribution factor m with known experimental measurements of maximum stresses. The data of gas holdup distributions are not provided in the source article (Moslemian et al., 1992); therefore, the average gas holdup is estimated using Eq. 22. Then, the calculated average gas holdups are substituted into Eq. 21 to yield values of m, as shown in Table 3. Equation 15 is then used to calculate the distribution of the dimensional

Table 2. Computed Maximum Reynolds Shear Stresses of Systems with Different Scales

Systems	U_g (cm/s)	R (m)	$\overline{\epsilon_g}$ Exp.	m Fitting	$\tau_{t,\max}$ Pa From Eq. 16	$\tau_{t,\max}$ Pa Exp.
Lab scale*	1.9	0.07	0.07	8	5.59	
	3.8	0.07	0.137	6	9.70	
	6.4	0.07	0.182	5	11.38	
	16.9	0.07	0.220	4	20.19	
Industrial scale**	2.2	2.75	0.0813	130	26.13	
	3.6	2.75	0.107	100	28.55	
	4.4	2.75	0.127	70	30.86	
Lab scale†	10.5	0.146	0.18	6	26.58	28.3

*From Hills (1974).
**From Kojima et al. (1980).
†From Devanathan et al. (1990).

Table 3. Predicted Gas Holdup Distribution Factors of a Lab Scale Column*

U_g (cm/s)	R (m)	$\overline{\epsilon_g}$ Eq. 22	$\tau_{t,\max}$, Pa Exp.	m Eq. 21
14	0.095	0.202	24.7	4.12
10	0.095	0.166	17.2	5.33
6	0.095	0.122	5.3	16.90
2	0.095	0.0607	1.9	25.12

*System is from Moslemian et al. (1992).

Reynolds stress with $\lambda \approx 1.0$, as shown in Figure 2. The agreement between the predicted and experimental values is fairly good even though the near-wall negative shear stresses have been neglected. This suggests that the maximum Reynolds shear stress can be used as an important characteristic parameter with which the local Reynolds shear stress may be evaluated. The analysis of this may be useful in biological applications of bubble columns. In the design of a two-phase bioreactor, a dilemma between increasing aeration to enhance growth of biomasses and avoiding damage of shear-sensitive cells has eluded process designers. Using the present analysis and corresponding relations, the local Reynolds shear stress, particularly the maximum shear stress, can be readily evaluated. Through this, making a compromise between effects of aeration and associated shear damages is straightforward.

Conclusions

A fresh dynamic analysis with the 1-D flow models is developed to make clear three main characteristic flow parameters, that is, the reversion point of the flow, radial position of the local maximum Reynolds shear stresses, and maximum reverse-flow position. Analytical relations for estimating the parameters and other relevant corollaries are derived. The validity of the theory is examined by a comparison of the model results with previous modeling work. It is found that the present analytical solutions are the general cases relative to previous ones. The extension and applications of the theory suggest that, the theory can reflect the main features of the buoyancy-driven two-phase flow in bubble columns within the restriction of the 1-D flow model and thereby has applicability and flexibility in modeling this type of flow.

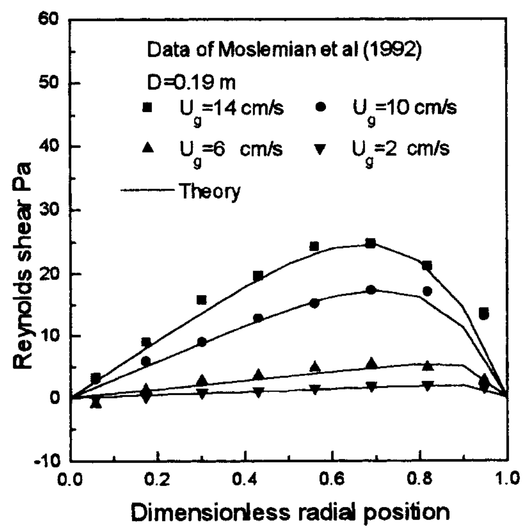

Figure 2. Reynolds shear stress: present model prediction vs. experimental data of Moslemian et al. (1992).

Notation

- Ar = Archimedes number, $(\rho_l^2 R^3 g \overline{\epsilon_g})/\mu_m^2$
- D = column diameter, m
- g = gravitational constant, m/s^2
- m = gas holdup distribution factor
- r = dimensional radial position, m
- R = column radius, m
- $T_{\max}$ = dimensionless maximum Reynolds shear stress
- U_g = superficial gas velocity, m/s
- β = dimensionless parameter, $(\mathrm{Re}_w^2/\mathrm{Ar})$
- δ = reversion point of flow
- ϵ_g = local gas holdup
- $\overline{\epsilon_g}$ = average gas holdup
- γ = maximum Reynolds shear stress point
- λ = maximum reverse-flow point
- μ_m = molecular viscosity, Pa·s
- ρ_l = liquid density, kg/m^3
- τ = shear stress of flow, Pa
- $\tau_{\max}$ = maximum Reynolds shear stress, Pa
- τ_w = wall shear stress, Pa
- ξ = dimensionless radial position

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